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Assuming that quasiparticle (QP) excitations near the nodes of d-wave superconductors are described by a relativistic four-fermion model, the QP thermal conductivity in the presence of an external magnetic field is calculated. It is shown that, for narrow width of quasiparticles, the thermal conductivity, as a function of the applied magnetic field, exhibits a kink behavior at a critical field $B_c \sim T^2$. The kink is due to the opening of a gap in the magnetic field at B_c and to the enhancement of the transitions between the zeroth and first Landau levels. The results are applied to explain Krishana et al. experiments in high-T_c compounds in the clean limit.

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It is now well known from the study of field theoretical models that an external magnetic field can be a strong catalyst for a symmetry breaking leading to the generation of a fermion dynamical mass even at the weakest attractive interactions [1]. This phenomenon of magnetic catalysis (MC) of symmetry breaking has been shown to be essentially model independent and to have wide applications in several physical systems [1–4], in particular, in condensed matter physics [5–7]. The essence of the MC effect lies in the dimensional reduction of the fermion pairing dynamics when the pairing energy is much less than the Landau gap $\sqrt{eB}$ (B is the magnitude of the magnetic field induction). In this case, the fermions are mostly confined to the lowest Landau level (LL) and their attractive interaction, regardless how small it may be, leads to mass generation. The lowest LL plays in this case a role similar to that of the Fermi surface in BCS superconductivity [1].

The magnetic catalysis may play a relevant role in the physics of high-T_c superconductors in external magnetic fields. To see this, let us recall that the quasiparticle (QP) excitations at the nodes of planar $d-$ wave superconductors can be described by a massless Dirac Lagrangian [8,9]. A natural candidate to modeling the QP interactions in a d-wave superconductor would be a 3-dimensional relativistic four-fermion (Nambu-Jona-Lasinio (NJL) type) model. It is well known [1,3] that the gap equations of such models in the presence of an external magnetic field lead to the generation of a dynamical fermion mass (QP gap), which scales (at zero temperature) with the field as $m \sim \sqrt{eB}$. The critical temperature at which the dynamical mass vanishes is determined by the dynamical mass at zero temperature and, thus, scales with the magnetic field in a way quite similar to the scaling law found in recent experiments on thermal conductivity in high-T_c compounds [10,11].

According to these experiments, at temperatures significantly lower than the critical temperature of superconductivity, the thermal conductivity κ , as a function of a magnetic field perpendicularly applied to the cuprate planes, displays a sharp break in its slope at a transition field B_c, followed then by a plateau region in which it ceases to change with increasing field up to the highest attainable fields $\sim 14T$. The critical temperature for the appearance of the kink-like behavior scales with the magnetic field as $T_c \sim \sqrt{B}$. Hence, the square root field dependence of the critical temperature in $3-$dimensional NJL models provides a sound indication that the MC phenomenon could be important to explain the experimental results of Krishana et al. [10].

By now, several possible mechanisms of arising plateau have been proposed [5–7,12–14]. All of them are based on the adoption of the QP picture as the low-energy excitations at the nodes of the $d-$wave symmetric order parameter. In the approach of Refs. [5–7,13] it is further supposed that a QP gap opens at the critical field B_c leading to the exponential vanishing of the quasiparticle contribution in the thermal conductivity. Since the total conductivity is assumed to be the sum of the QP term $\kappa_{\rm QP}$ and the phonon term $\kappa_{\rm ph}$ with the phononic part independent of the field, this would lead to a plateau for high fields. Laughlin [13] relates the QP gap to a weakly first order phase transition leading to the development of an additional small parity-violating superconducting order parameter. However, as was mentioned in [14], sudden opening of a gap would lead to a jump in κ opposite to the kink observed in [10,11]. In another line of reasoning [5–7,12], the drop in the QP's conductivity, hence the plateau in the total conductivity, is ascribed to the opening of a field-induced energy gap in d-wave superconductors due to a second order phase transition.

On the contrary, the approach of Franz [14] does not appeal to the generation of any QP gap. Albeit the phononic part of the thermal conductivity is also considered to be field-independent, in Franz' scenario the QPs conductivity itself becomes field-independent above a crossover field B_c. The effect is primarily due to the compensation of the enhancement of the QP density of states, arising from the Doppler shift of QP's spectrum in the superfluid velocity field around vortices (Volovik effect [15]), and the growth of QP's width caused by the scattering on vortices. The Volovik effect certainly plays a decisive role at weak magnetic fields and low temperatures (less than 1 K), where the increase in the thermal conductivity was predicted theoretically [16], and later confirmed in the experiment [11], to follow the $\sqrt{B}$ dependence of the density of states. However, Volovik's arguments are based on semiclassical calculations valid in case one has well isolated vortices with supercurrents circulating around them. At higher fields, when vortices begin to overlap, this semiclassical picture should be replaced by a more adequate quantum mechanical treatment where the Landau level quantization could play an essential role. It was argued by Anderson [9] that the approximation of constant magnetic field inside high-T_c superconductors may be rather reasonable for the description of the QP's transport in the range of fields $H_{c_1} \ll B \ll H_{c_2}$ (H_{c_1}, H_{c_2} are lower and upper critical magnetic fields, correspondingly) where the vortices are dense enough to overlap strongly. Though the issue of LL quantization is still debated [9,17], we think it is worth to elaborate the consequences of this assumption.

It should be stressed that none of the above works address the question of the kink itself in the thermal conductivity. The aim of the present paper is to discuss a mechanism that generates the kink effect within the framework of the MC phenomenon. We present for the first time a consistent calculation of the thermal conductivity in the presence of an external magnetic field in a model with the simplest four-fermion interaction for quasiparticles. We use the same constant magnetic field approximation that was already explored in Ref. [6,18] to calculate the thermal conductivity. However, our calculation deviates considerably from what was done in [6,18]. Not only we take into account the contribution of all Landau levels, but the definition of the heat current itself is different. In the limit of narrow width of quasiparticles ($\Gamma \ll T, \sqrt{B}$), after a gap is opened, the thermal conductivity exhibits a new term proportional to σ^2 (σ is the gap). This term originates from the compensation of the matrix elements of transitions between the zeroth and the first LLs (behaving as $1/B$) and the LL density of states which in turn is proportional to B. This is one more manifestation of the MC phenomenon: not only a gap is induced by the magnetic field, but the transitions between the zeroth and the first LLs are enhanced. In mean-field approximation and near the phase transition point the gap behaves like $\sigma \simeq 0.523\sqrt{eB - eB_c}$ what yields a positive contribution into the slope of the thermal conductivity leading to a jump (kink effect) of $\kappa(B)$ at $B = B_c$.

We start from the following NJL model in an external magnetic field in $2+1$ dimensions, which is believed to possess the symmetries of the $d-$wave hamiltonians defined on a lattice [6]

$$\mathcal{L} = \bar{\psi}_i[i\gamma^0\hbar\frac{\partial}{\partial t} + iv_D\gamma^i(\hbar\frac{\partial}{\partial x^i} - \frac{e}{c}A_i)]\psi + \frac{g}{2N}(\bar{\psi}_i\psi_i)^2, \tag{1}$$

In (1) the vector potential is taken in the symmetric gauge $A_\mu = \left(0, -\frac{B}{2}x_2, \frac{B}{2}x_1\right)$, $v_D = \sqrt{v_F v_\Delta}$ with v_F, v_Δ being the velocities perpendicular and tangential to the Fermi surface respectively. They originate from the quasiparticle excitation spectrum in the vicinity of the gap nodes that takes the form of an anisotropic Dirac cone $E(k) = \sqrt{v_F^2 k_1^2 + v_\Delta^2 k_2^2}$. After rescaling coordinates this leads to Eq. (1). In what follows we take $\hbar = v_D = k_B = 1$ and absorb c into the charge e. We will restore these constants when needed. We assume also that the fermions carry an additional flavor index $i = 1, \ldots, N$ ($N = 2$ for realistic d-wave superconductors). The Dirac γ matrices are taken in a reducible four-component representation.

The Lagrangian density (1) is invariant under the discrete (chiral) symmetry $\psi \to \gamma_5\psi$, $\bar{\psi} \to -\bar{\psi}\gamma_5$, which forbids the fermion mass generation in perturbation theory. The mass generation can be studied introducing an auxiliary field $\sigma = -(g/N)\bar{\psi}_i\psi_i$ by means of the Hubbard-Stratanovich trick which permits one to integrate over fermion fields in the path integral representation of the partition function. The field σ has no dynamics at the tree level but it acquires the kinetic term due to fermion loops. Likewise, a nontrivial minimum of the effective potential (the expectation value of σ) gives mass to fermions and spontaneously breaks the discrete symmetry leading to a neutral condensate of fermion-antifermion pairs. Studying the minimum of the effective potential we find that, at a fixed temperature T, there is a critical value of the magnetic field $\sqrt{eB_c}/T \simeq 4.148$ such that for subcritical fields $eB \le eB_c$ the gap is zero, while for $eB > eB_c$ a nontrivial gap appears. The critical curve equation has the form $(v_D/c)^2 10^{10} B = 21.5 \cdot T^2$ for B measured in Tesla. Using the estimated value of the characteristic velocity $v_D = 1.16 \times 10^7 cm/s$ reported in [7], we obtain the critical curve $B = 0.014 \cdot T^2$, which fits the experimental curve of Ref. [10].

To derive an expression for the static thermal conductivity in an isotropic system, we follow the familiar linear response method and apply Kubo's formula [19]

$$\kappa = -\frac{1}{TV}\mathrm{Im}\int_0^\infty dtt \int d^2x_1 d^2x_2 \langle u_i(x_1, 0) u_i(x_2, t)\rangle, \tag{2}$$

where V is the volume of the system and $u_i(x,t)$ is the heat-current density operator. The brackets denote averaging in the canonical ensemble. Physically, the thermal conductivity κ appears as a coefficient in the equation relating the heat current to the temperature gradient $\vec{u} = -\kappa\vec{\nabla}T$ under the condition of absence of particle flow. If we neglect the chemical potential, the heat density coincides with the energy density ϵ, hence the quantity $\vec{u}$ that satisfies the continuity equation $\dot{\epsilon}(x) + \vec{\nabla}\vec{u}(x) = 0$ can be interpreted as the heat current. From the Lagrangian density (1) we find $u_i = \frac{i}{2}\left(\bar\psi\gamma_i\partial_0\psi - \partial_0\bar\psi\gamma_i\psi\right)$.

Neglecting vertex corrections [20] the calculation of the thermal conductivity reduces to the evaluation of the bubble diagram [21]. In this approximation, making use of the spectral representation for the fermion Green's function, we arrive at the following expression for the thermal conductivity in the Matsubara formalism

$$\kappa = \frac{1}{32\pi T^2}\int_{-\infty}^{\infty}\frac{d\omega\omega^2}{\cosh^2\frac{\omega}{2T}}\int d^2k\,\mathrm{tr}\Big[\gamma^i A(\omega,\vec{k})\gamma^i A(\omega,\vec{k})\Big]\,. \quad (3)$$

The spectral function $A(\omega,\vec{k}) = -\pi^{-1}\mathrm{Im}S^R(\omega+i\varepsilon,\vec{k})$ is derived from the fermion propagator in a magnetic field decomposed over Landau level poles [1,22],

$$A(\omega,\vec{k}) = e^{-\frac{\vec{k}^2}{eB}}\frac{\Gamma}{2\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{M_n}\left[\frac{(\gamma^0 M_n+\sigma)f_1(\vec{k})+f_2(\vec{k})}{(\omega-M_n)^2+\Gamma^2}\right.$$
$$\left.+\frac{(\gamma^0 M_n-\sigma)f_1(\vec{k})-f_2(\vec{k})}{(\omega+M_n)^2+\Gamma^2}\right],\; M_n=\sqrt{\sigma^2+2eBn}, \quad (4)$$
$$f_1(\vec{k}) = 2\left[P_-L_n(u)-P_+L_{n-1}(u)\right],\, f_2(\vec{k}) = 4\vec{k}\vec{\gamma}L^1_{n-1}(u).$$

Here $P_\pm = (1\pm i\gamma^1\gamma^2)/2$ are projector operators, L_n, L_n^1 are Laguerre's polynomials ($L^1_{-1}=0$), $u = 2\vec{k}^2/eB$, and σ is the dynamical fermion mass obtained from the finite temperature gap equation in the magnetic field. We introduced also the quasiparticles width replacing ε in the definition of $A(\omega,\vec{k})$ by finite Γ, which is due to interaction processes, in particular, scatterings on impurities. Straightforward calculation of the trace and further integration over momenta in Eq. (3) produce Kronecker's delta symbols $\delta_{n,m-1}+\delta_{m,n-1}$ due to orthogonality of Laguerre's polynomials. This implies that only those transitions between neighbor Landau levels contribute to the heat transfer. After performing one of the sums in (3) we obtain

$$\kappa = \frac{eB\Gamma^2N}{\pi^2T^2}\sum_{n=0}^{\infty}\int_0^{\infty}\frac{d\omega\omega^2}{\cosh^2\frac{\omega}{2T}}\frac{1}{(\omega^2+M_n^2+\Gamma^2)^2-4\omega^2M_n^2}$$
$$\times\frac{(\omega^2+M_n^2+\Gamma^2)(\omega^2+M_{n+1}^2+\Gamma^2)-4\omega^2\sigma^2}{[(\omega^2+M_{n+1}^2+\Gamma^2)^2-4\omega^2M_{n+1}^2]}. \quad (5)$$

Further summation over n in Eq. (5) can be carried out expanding the integrand in terms of partial fractions. The resulting sums are expressed through digamma functions and the final expression for κ is written as follows

$$\kappa = \frac{N\Gamma^2}{2\pi^2T^2}\int_0^{\infty}\frac{d\omega\omega^2}{\cosh^2\frac{\omega}{2T}}\frac{1}{(eB)^2+(2\omega\Gamma)^2}\Big\{2\omega^2+$$
$$\frac{(\omega^2+\sigma^2+\Gamma^2)(eB)^2-2\omega^2(\omega^2-\sigma^2+\Gamma^2)eB}{(\omega^2-\sigma^2-\Gamma^2)^2+4\omega^2\Gamma^2}-$$
$$\frac{\omega(\omega^2-\sigma^2+\Gamma^2)}{\Gamma}\mathrm{Im}\psi\left(\frac{\sigma^2+\Gamma^2-\omega^2-2i\omega\Gamma}{2eB}\right)\Big\}. \quad (6)$$

This formula is the main result of our paper and we can use it now to study the different asymptotic regimes. It is easy to show that in the limit of zero field κ reduces to

$$\kappa_0 = \frac{N}{4\pi^2T^2}\int_0^{\infty}\frac{d\omega\omega^2}{\cosh^2\frac{\omega}{2T}}\left[1+\frac{\omega^2+\Gamma^2}{\omega\Gamma}\arctan\frac{\omega}{\Gamma}\right], \quad (7)$$

where we put $\sigma = 0$, since the mass is not generated in the zero-field weak coupling NJL model. Eq. (7), up to an overall factor $k_B^2(v_F^2+v_\Delta^2)/\hbar v_F v_\Delta$, coincides with the corresponding expression obtained in Ref. [14]. With the overall factor replacing N ($=2$ in real $d-$wave superconductor), Eq. (7) reproduces the universal (or residual) thermal conductivity at low T in the so-called "dirty" limit, $T \ll \Gamma$ [20]. The residual conductivity was recently observed in experiments [23] giving explicit confirmation of the existence of gapless quasiparticles in d-wave cuprates at $T < T_c$.

Let us consider the case $\Gamma \ll T$ with $B \neq 0$. In this approximation we obtain

$$\kappa \simeq \frac{N\Gamma}{4\pi T^2}\left\{\frac{\sigma^2}{\cosh^2\frac{\sigma}{2T}}+4\sum_{n=1}^{\infty}\frac{n(\sigma^2+2eBn)}{\cosh^2\frac{\sqrt{\sigma^2+2eBn}}{2T}}\right\}. \quad (8)$$

Asymptotically, at $\sqrt{eB} \gg \sqrt{eB_c} \simeq T$, the dynamical mass behaves as $\sigma \sim \sqrt{eB}$ what leads to an exponential decrease as in the case of gapless fermions. However, the term $\cosh^{-2}\sigma/2T$ in Eq. (8) is of order one when $\sigma \lesssim 2T$, thus, there is no suppression of this term for a certain range of fields where it is almost field independent (plateau region).

Near the phase transition point $\sigma \simeq a\sqrt{eB-eB_c}$ and the first term in Eq. (8) gives positive contribution to the slope of the thermal conductivity at $eB \geq eB_c$ leading to the jump in the slope of κ (kink effect) at $eB = eB_c$. The parameter a is model-dependent. For the NJL model (1) we find, in mean-field approximation, $a \simeq 0.523$.

The explicit appearance of the kink has been corroborated by numerical calculations as shown in Fig.1. Notice the break in the slope of κ (kink effect) at the critical value B_c in the presence of σ. For $B > B_c$ the kink is followed by a region where κ is only weakly dependent on the field. While decreasing the temperature, the position of the kink moves to the left in accordance with the critical line $B_c = 0.014T^2$. Fig. 1 has one more similarity with the experimental behavior reported in [10]: with decreasing T the crossing of the curves occurs in such a way that the lower T curve reaches the higher value at large fields.

Because of the important role played by the first term in (8) to get the kink effect, it is instructive to clarify its origin. From Eq. (5) one can see that while the contribution of transitions between Landau levels with $n \geq 1$ in the integrand behaves as $1/(eB)^2$ for large fields, the contribution of transitions between the zeroth and the first LLs decreases only as the first power of the field ($\sim 1/eB$). Since the density of LLs is proportional to

eB, this implies that the transitions between the zeroth and the first LLs are not suppressed, even though the gap between the levels grows with the field. Note that for gapless QPs the transitions with $n = 0$ are completely suppressed in the regime $\Gamma \to 0$: indeed, their contribution is proportional to $\delta(\omega)$ and the integrand in (5) contains an ω^2 factor. For gapped QPs, $\delta(\omega)$ is replaced by $\delta(\omega \pm \sigma)$ and the transitions with $n = 0$ survive.

In summary, in the present paper the thermal conductivity at $B \neq 0$ in the framework of a relativistic four-fermion model used to describe QP's interactions in $d-$wave cuprates was calculated. Assuming that the approximation of a uniform magnetic field inside $d-$wave superconductors is suitable for the case of overlapping vortices, we showed that the thermal conductivity exhibits a kink behavior when the field reaches the critical value $B_c = 0.014 \cdot T^2$. Two main features determine this effect: the generation of a QP gap in a magnetic field (MC phenomenon) and the lack of suppression of zeroth-first LLs transitions.

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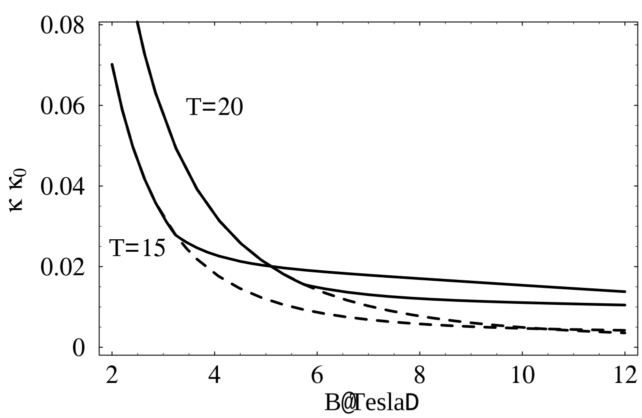

FIG. 1. The magnetic field dependence of κ at $T = 20K$ and $T = 15K$ in the narrow width case ($\Gamma = 5K$). The solid lines represent κ/κ_0 when a QP gap σ is MC-induced at $B \geq B_c(T)$ ($B_c(20) = 5.75$T, $B_c(15) = 3.23$T). The dashed lines represent the behavior of κ/κ_0 when σ remains zero at $B \geq B_c(T)$.